MATHEMATICS SPECIALIST

MAWA Year 12 Examination 2018

Calculator-assumed

Marking Key

The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2018**

Question 8 **(5 marks)**

Solution
$\lvert 4-3x\rvert = 5+\lvert x+2\rvert$ Critical values $x=-2,\ x=\frac{4}{3}$ $x\le -2 \quad \Rightarrow \quad 4-3x=5-x-2$ $\Rightarrow \quad 2x=1$ $x=\frac{1}{2}$ Contradiction $-2<x<\frac{4}{3} \quad \Rightarrow \quad 4-3x=5+x+2$ $\Rightarrow \quad 4x=-3$ $\Rightarrow \quad x=-\frac{3}{4}$ $x\ge\frac{4}{3} \quad \Rightarrow \quad -4+3x=5+x+2$ $2x=11$ $x=\frac{11}{2}$ Solution: $x=-\frac{3}{4},\ x=\frac{11}{2}$

Mathematical behaviours	Marks
• states the correct critical values	1
• writes correct equations for each sub-interval	1
• determines correct conclusion for $x\le -2$	1
• correct solution for $-2<x<\frac{4}{3}$	1
• provides the correct solution for $x\ge\frac{4}{3}$	1

Question 9 (a) **(3 marks)**

Solution	
If $$\frac{1}{w}=\frac{1}{1-i}+\frac{2}{i}=\frac{2-i}{(1-i)i}$$ then $$w=\frac{i(1-i)}{2-i}=\frac{(1+i)}{2-i}\times\frac{2+i}{2+i}=\frac{1}{5}(1+3i)$$	
Mathematical behaviours	Marks
• combines the two fractions • inverts the expression and multiplies by the complex conjugate • deduces the correct real and imaginary parts	1 1 1

Question 9(b) **(1 mark)**

Solution	
From calculator $i^i \approx 0.20787958$	
Mathematical behaviours	Marks
• states the value required quoting the answer to eight places	1

Question 9(c) **(1 mark)**

Solution	
Since $\exp(i\beta)=\cos\beta+i\sin\beta=i$ if $\cos\beta=0$ and $\sin\beta=1$. The smallest possible solution is clearly $\beta=\pi/2$.	
Mathematical behaviours	Marks
• solves for the required β giving the answer in radians	1

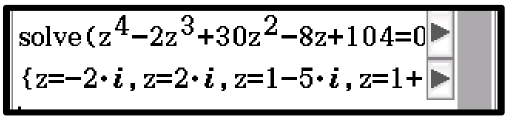
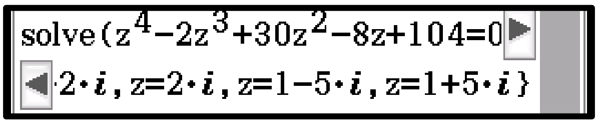
Question 9(d) **(1 mark)**

Solution	
From (b) we deduce that $i^i=[\exp(i\pi/2)]^i=\exp(i^2\pi/2)=\exp(-\pi/2)$.	
Mathematical behaviours	Marks
• combines the results of parts (a) and (b) to infer the exact value	1

Question 10(a) **(2 marks)**

Solution	
Given that $z = 2i$ is a solution of $z^4 - 2z^3 + mz^2 + nz + 104 = 0$ then $16 + 16i - 4m + 2in + 104 = 0$ Real parts give that $120 - 4m = 0 \quad \Rightarrow \quad m = 30$ and imaginary parts imply that $16 + 2n = 0 \;\Rightarrow\; n = -8.$	
Mathematical behaviours	**Marks**
• substitutes the value $z = 2i$ into the equation • deduces the required values of m and n	1 1

Question 10(b) **(4 marks)**

Solution	
Since $z = 2i$ is a solution of the polynomial equation then so is $z = -2i$ Hence the quartic has a factor $(z - 2i)(z + 2i) = (z^2 + 4)$ By long division $$z^4 - 2z^3 + 30z^2 - 8z + 104 = (z^2 + 4)(z^2 - 2z + 26)$$ By the quadratic formula the equation $z^2 - 2z + 26 = 0$ has solutions $z = 1 \pm 5i$ Hence the required three other solutions are $-2i$ and $1 \pm 5i$ 	
Mathematical behaviours	**Marks**
• correctly notes that the conjugate $z = -2i$ is also a solution • notes that the quartic has a factor $(z^2 + 4)$ • factorises the quartic • deduces the required extra solutions of the quartic	1 1 1 1

Question 11(a) **(1 mark)**

Solution

$\boldsymbol{a} = (2\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}) \times (3\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k})$
$= (3 \times 1 - 2 \times 5)\boldsymbol{i} + (2 \times 3 - 1 \times 2)\boldsymbol{j} + (2 \times 5 - 3 \times 3)\boldsymbol{k} = -7\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$

```
[2,3,2]⇒a
                    [2 3 2]
[3,5,1]⇒b
                    [3 5 1]
crossP(a,b)
                   [-7 4 1]
```

Mathematical behaviours	Marks
• evaluates the cross product correctly	1

Question 11(b) **(2 marks)**

Solution

$2\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ and $3\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$ are normal to the planes $\mathcal{P}_1$ and $\mathcal{P}_2$
and $\boldsymbol{a}$ is perpendicular to both of these normals. (*)
So $\boldsymbol{a}$ is parallel to vectors in each of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$
So $\boldsymbol{a}$ is parallel to the line of intersection of these planes, i.e. L. (**)

Mathematical behaviours	Marks
• obtains result (*) • completes the argument correctly (**)	1 1

Question 11(c) **(2 marks)**

Solution

Since $\boldsymbol{a}$ is parallel to L, a vector equation for L has the form $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} = \boldsymbol{r_0} + t\boldsymbol{a}$

As $(0,1,0)$ lies in L, we may assume that $\boldsymbol{r_0} = \boldsymbol{j}$

So $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} = \boldsymbol{j} + t(-7\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k})$ is a vector equation for L.

Mathematical behaviours	Marks
• evaluates $\boldsymbol{r_0}$ • obtains a correct vector form of equation	1 1

Question 11(d)(i) **(3 marks)**

Solution	
The equations $\begin{matrix} 2x+3y+2z=3 \\ 3x+5y+z=5 \\ 7x+11y+az=b \end{matrix}$ reduce to $\begin{matrix} x+7z=0 \\ y-4z=1 \\ (a-5)z=b-11 \end{matrix}$ (*) The planes $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ have infinitely points in common if the third equation reduces to $0 \times z = 0$, i.e. if $a = 5$ and $b = 11$	
Mathematical behaviours	**Marks**
• attempts simultaneous reduction of the equations for $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$	1
• derives equation (*)	1
• deduces the correct solutions	1

Question 11(d)(ii) **(1 mark)**

Solution	
The planes $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ have no point in common if equations (*) in d(i) are inconsistent, i.e. the third equation reduces to $0 \times z \neq 0$, i.e. if $a = 5$ and $b \neq 11$.	
Mathematical behaviours	**Marks**
• deduces the correct solution	1

Question 12 (a) **(2 marks)**

Solution	
$f \circ g(x) = 2\cos\sqrt{1-x}$ $g \circ f(x) = \sqrt{1-2\cos x}$	
Mathematical behaviours	**Marks**
• derives correct expression for $f \circ g(x)$ • derives correct expression for $g \circ f(x)$	1 1

Question 12 (b) **(2 marks)**

Solution	
Domain $f \circ g = \{x : 1-x \geq 0,\ x \in \mathbb{R}\}$ $= \{x : x \leq 1,\ x \in \mathbb{R}\}$ Range $f \circ g = \{y : -2 \leq y \leq 2,\ y \in \mathbb{R}\}$	
Mathematical behaviours	**Marks**
• states correct domain • states correct range	1 1

Question 12 (c) **(3 marks)**

Solution	
Domain $g \circ f = \left\{x : \cos x \leq \frac{1}{2},\ x \in \mathbb{R}\right\}$ $= \left\{x : 2k\pi + \frac{\pi}{3} \leq x \leq 2k\pi + \frac{5\pi}{3},\ k \text{ an integer},\ x \in \mathbb{R}\right\}$ Range $g \circ f = \left\{y : 0 \leq y \leq \sqrt{3},\ y \in \mathbb{R}\right\}$	
Mathematical behaviours	**Marks**
• states correct domain including mention of the $\pi / 3$ • states correct range	1+1 1

Question 13(a) **(3 marks)**

Solution

For $0 \le x \le \dfrac{\pi}{4}$ the curve lies above the line so

$$A = \sqrt{2}\int_0^{\pi/4} \cos x\,dx - \frac{4}{\pi}\int_0^{\pi/4} x\,dx = \sqrt{2}\left[\sin x\right]_0^{\pi/4} - \frac{2}{\pi}\left[x^2\right]_0^{\pi/4} = 1 - \frac{\pi}{8}$$

$$\sqrt{2}\int_0^{\pi/4}\cos(x)\,dx - \int_0^{\pi/4}\frac{4x}{\pi}\,dx$$

$$\frac{-\pi}{8}+1$$

Mathematical behaviours	Marks
• gives a sketch of the form of the area	1
• writes down the correct expression for the required area	1
• evaluates the two integrals to deduce the necessary result	1

Question 13(b) **(2 marks)**

Solution

Required volume of rotation is given by

$$V = \pi\int_0^{\pi/4} (\sqrt{2}\cos x)^2\,dx - \pi\int_0^{\pi/4}\left(\frac{4}{\pi}x\right)^2 dx = \pi\int_0^{\pi/4}(1+\cos 2x)\,dx - \frac{16}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi/4}$$

$$= \pi\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/4} - \frac{16}{\pi}\times\frac{\pi^3}{192} = \pi\left[\frac{\pi}{4}+\frac{1}{2}\right] - \frac{\pi^2}{12} = \frac{1}{6}\pi^2 + \frac{1}{2}\pi$$

Mathematical behaviours	Marks
• writes down the appropriate expression for the volume for the two parts	1
• deduces the correct volume	1

Question 14 (a) **(2 marks)**

Solution	
Using $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(\frac{1}{2}(25-5x^2)\right) = -5x$ Since $a = -\left(\sqrt{5}\right)^2 x$, then motion is SHM with period = $\frac{2\pi}{\sqrt{5}}$.	
Mathematical behaviours	**Marks**
• differentiates to obtain experession for acceleration • shows motion is simple harmonic with correct period	1 1

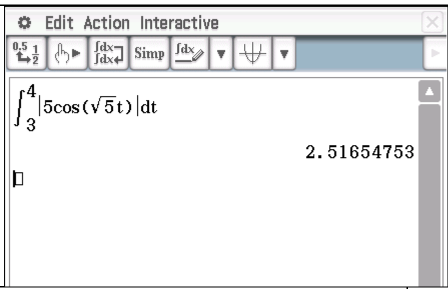
Question 14 (b) **(2 marks)**

Solution	
$v^2 = 5\left(5 - x^2\right)$ $x = \sqrt{5}\sin\left(\sqrt{5}\,t\right)$	
Mathematical behaviours	**Marks**
• states correct amplitude • states correct displacement equation	1 1

Question 14 (c) **(2 marks)**

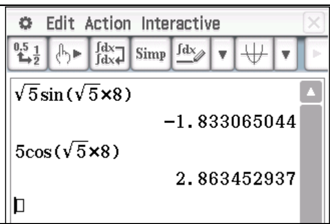
Solution	
max speed $= 5$ cms^{-1} min speed $= 0$ cms^{-1}	
Mathematical behaviours	**Marks**
• states correct maximum speed • states correct minimum speed	1 1

Question 14 (d) **(3 marks)**

Solution
$v(t) = 5\cos(\sqrt{5}t)$ distance $= \int_3^4 \lvert v(t) \rvert \, dt$ $= \int_3^4 \lvert 5\cos(\sqrt{5}t) \rvert \, dt$ $= 2.52$ cm to 2 dp 

Mathematical behaviours	Marks
• expresses velocity as a function of time	1
• integrates using correct upper and lower values	1
• calculates distance travelled	1

Question 14 (e) **(2 marks)**

Solution
Since $x(8) < 0$ and $v(8) > 0$, the particle is travelling towards the origin when $t = 8$. 

Mathematical behaviours	Marks
• calculates $x(8)$ and $v(8)$	1
• draws correct conclusion	1

Question 15 **(4 marks)**

Solution
If we write $\dfrac{x+3}{x(x-1)} = \dfrac{A}{x} + \dfrac{B}{x-1} = \dfrac{(A+B)x - A}{x(x-1)}$ Equating the coefficients gives $A = -3$ and $B = 4$ Hence $\displaystyle\int_2^4 \frac{x+3}{x(x-1)}\,dx = -3\int_2^4 \frac{dx}{x} + 4\int_2^4 \frac{dx}{x-1} = \left[-3\ln x + 4\ln(x-1)\right]_2^4 = 4\ln 3 - 3\ln 4 + 3\ln 2$ $= \ln\left(\dfrac{3^4 \times 2^3}{4^3}\right) = \ln\left(\dfrac{81}{8}\right)$

Mathematical behaviours	Marks
• writes down the appropriate form of the partial fractions	1
• compares coefficients to deduce the constants A and B	1
• integrates correctly	1
• deduces the required result	1

Question 16 (a) **(1 mark)**

Solution	
The parabola joins the orgin with the point (9,6) Line cuts parabola within the domain if $9a > 6 \Rightarrow a > 2/3$.	
Mathematical behaviours	Marks
• determines correctly the required inequality	1

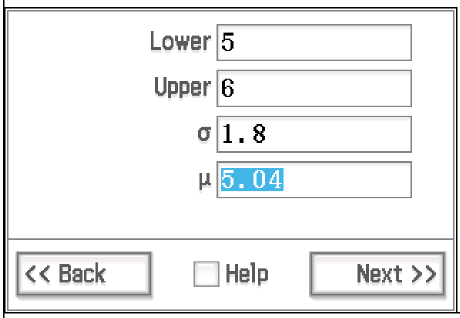
Question 16 (b) **(3 marks)**

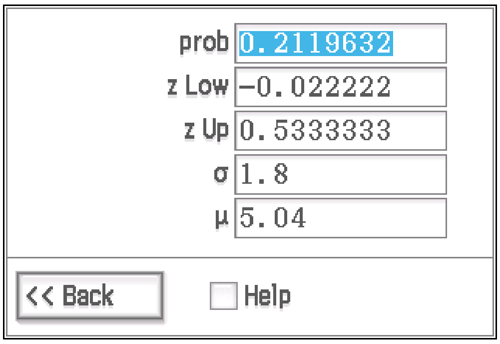
Solution	
There is no need to compute the point of intersection. The two areas are equal if the area under the parabola over $0 \le x \le 9$ matches the area under the line. Now $\int_0^9 2\sqrt{x}dx = \frac{4}{3}\left[x^{3/2}\right]_0^9 = \frac{4}{3} \times 27 = 36$ and $\int_0^9 ax\,dx = \frac{1}{2}a\left[x^2\right]_0^9 = \frac{81}{2}a$ Thus $a = 72/81 = 8/9$	
Mathematical behaviours	Marks
• realises there is no need to determine the point of intersection • evaluates the areas under the line and parabola over $0 \le x \le 9$ • deduces the requisite value of a	1 1 1

Question 17(a) **(2 marks)**

Solution

$P(5 < X < 6) = P\left(\frac{5-5.04}{1.8} < \frac{X-5.04}{1.8} < \frac{6-5.04}{1.8}\right)$
$= P(-0.02 < Z < 0.53)$ (*)
$= 0.702 - 0.492 = 0.21$





Mathematical behaviours	Marks
• derives the correct Z-limits • obtains the correct answer	1 1

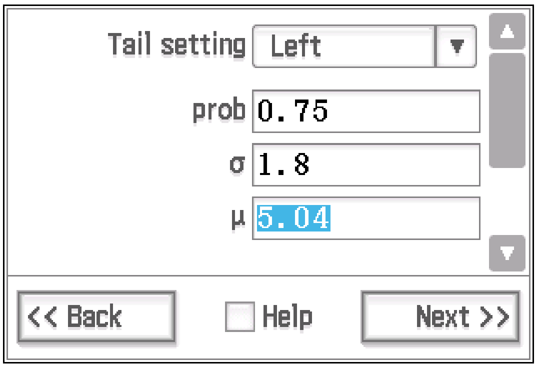
Question 17(b) **(2 marks)**

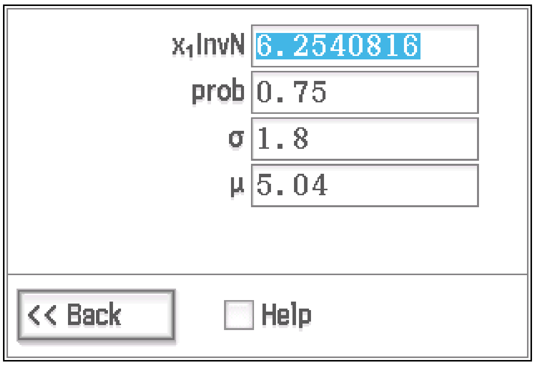
Solution

From the calculator (or tables) $P(Z < 0.675) = 0.75$

$$\frac{X - 5.04}{1.8} = 0.675 \Rightarrow X = 5.04 + 1.8 \times 0.675 = 6.255$$

That is, the upper quartile of the distribution is 6.255





Mathematical behaviours	Marks
• calculates 0.675 as the upper quartile in the normal distribution	1
• obtains the correct answer	1

Question 17(c) **(2 marks)**

Solution

From the formula sheet $E = z_\alpha \frac{\sigma}{\sqrt{n}}$ (*)

Since $z_{0.95} = 1.96$, $E = 0.2$ and $\sigma \cong 1.3$

$\sqrt{n} \cong 1.96 \times \frac{1.3}{0.2} = 12.74$ and so $n \cong 162.3$

So the sample size needs to be at least 163

Mathematical behaviours	Marks
• uses the formula (*)	1
• obtains the correct answer	1

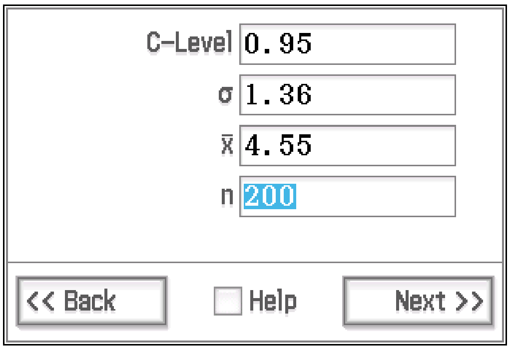
Question 17(d) **(2 marks)**

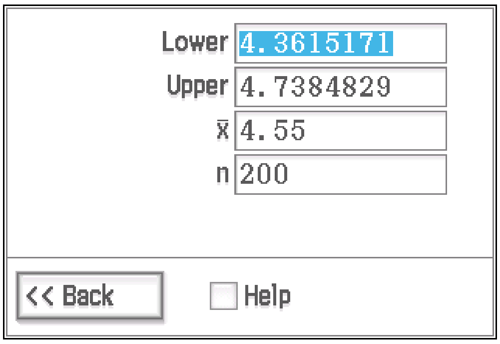
Solution

$\bar{X} = 4.55$, and $E = z_\alpha \frac{s}{\sqrt{n}} = 1.96 \times \frac{1.36}{\sqrt{200}} \cong 0.19$

So the 95% confidence interval is $4.55 - 0.19 < \mu_{acu} < 4.55 + 0.19$

i.e. $4.36 < \mu_{acu} < 4.74$





Mathematical behaviours	Marks
• evaluates E correctly	1
• dervies the correct limits for the confidence interval	1

Question 17(e) **(2 marks)**

Solution

The statement is correct.

This is because 5.04, the mean number of migraine attacks for untreated people, is much greater than 4.74, the upper limit of the upper limit of the 95% confidence interval for μ_{acu}, the average number of migraine attacks for people given acupuncture.

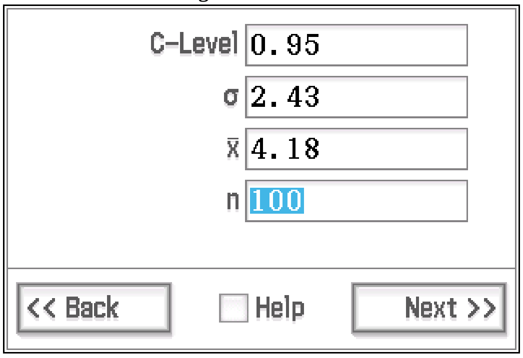
Mathematical behaviours	Marks
• accepts the claim	1
• gives a valid reason based on the confidence interval	1

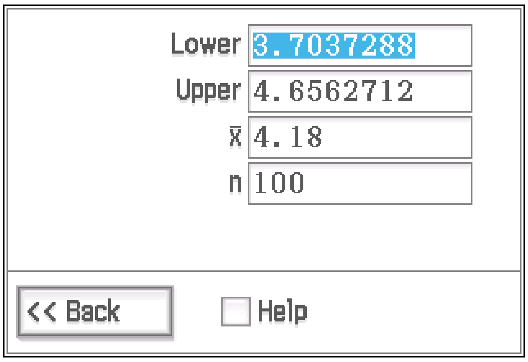
Question 17(f) **(5 marks)**

Solution

We determine a 95% confidence interval for μ_{drug}, the average number of migraine attacks for people who take the drug.

$\bar{X} = 4.18$, and $E = z_{\alpha}\frac{s}{\sqrt{n}} = 1.96 \times \frac{2.43}{\sqrt{100}} \cong 0.48$
So the 95% confidence interval is $4.18 - 0.48 < \mu_{drug} < 4.18 + 0.48$
i.e. $3.70 < \mu_{drug} < 4.66$





These tests DO NOT clearly show that the drug is more effective than acupuncture in treating migraine attacks, (*) because $4.36 < \mu_{acu} < 4.74$ and $3.70 < \mu_{drug} < 4.66$, the confidence intervals for μ_{acu} and μ_{drug}, overlap considerably.

Mathematical behaviours	Marks
• constructs a confidence interval for μ_{drug}	1
• sets the confidence level at least as high as 90%	1
• obtains correct limits for the confidence interval for μ_{drug}	1
• obtains the correct conclusion about the tests (*)	1
• gives a valid reason in terms of overlapping confidence intervals	1

Question 18(a) **(4 marks)**

Solution	
$\boldsymbol{v_1}(t) = \boldsymbol{v_0}(t) + \boldsymbol{a}t = 40\boldsymbol{i} + 20\boldsymbol{j} + (80 - 8t)\boldsymbol{k}$ and $\boldsymbol{r_1}(t) = \int_0^t \boldsymbol{v_1}(u)du = 40t\boldsymbol{i} + 20t\boldsymbol{j} + (80t - 4t^2)\boldsymbol{k}$, (*) because $\boldsymbol{r_1}(0) = \boldsymbol{0}$ $80t - 4t^2 = 0$ when $t = 0$ or $t = 20$. So rocket A returns to the horizontal plane H when $t = 20$. $\boldsymbol{r_1}(20) = \int_0^t \boldsymbol{v_1}(u)du = 800\boldsymbol{i} + 400\boldsymbol{j}$, and the distance of this point from O is $\sqrt{800^2 + 400^2} \cong 894\ m$	
Mathematical behaviours	**Marks**
• obtains expression for $\boldsymbol{v_1}(t)$ (*) • obtains expression for $\boldsymbol{r_1}(t)$ (*) • solves for t • obtains the correct answer	1 1 1 1

Question 18(b) **(3 marks)**

Solution	
Since $\boldsymbol{v_2}$ is constant, $\boldsymbol{r_2}(t) = \boldsymbol{r_2}(0) + t\boldsymbol{v_2} = 600\boldsymbol{i} + vt\cos\theta\boldsymbol{j} + vt\sin\theta\,\boldsymbol{k}$ (*) If the flight paths meet, $\boldsymbol{r_1}(t) = \boldsymbol{r_2}(t')$ for some values of t and t'. So $40t = 600$, $t = 15$ and $\boldsymbol{r_1}(15) = 600\boldsymbol{i} + 300\boldsymbol{j} + 300\boldsymbol{k}$ (**) If $\boldsymbol{r_2}(t') = 600\boldsymbol{i} + 300\boldsymbol{j} + 300\boldsymbol{k}$, $vt'\cos\theta = 300$ and $vt'\sin\theta = 300$. So $\cos\theta = \sin\theta$ and hence $\theta = 45°$.	
Mathematical behaviours	**Marks**
• solves for $\boldsymbol{r_2}(t)$ (*) • locates points where the flight paths meet • obtains correct answer for θ	1 1 1

Question 18(c) **(2 marks)**

Solution	
From part (b), if the rockets collide $\boldsymbol{r_2}(15) = 600\boldsymbol{i} + 300\boldsymbol{j} + 300\boldsymbol{k}$ (*) Also from (b), $t = 15$, and $\theta = 45°$ so that $15v\cos\theta = 300$ implying that $v = 20\sqrt{2} \cong 28.3$ So the speed of rocket B is $28\ ms^{-1}$ (approximately) if the rockets collide.	
Mathematical behaviours	**Marks**
• uses $\boldsymbol{r_2}(15) = \boldsymbol{r_1}(15)$ (*) • obtains correct answer	1 1

Question 19 **(7 marks)**

Solution
If the cross section of the sphere is given by $x^2 + y^2 = a^2$ then the volume of water is given by $$V = \pi \int_{y=-a}^{y=-a+b} x^2 \; dy = \pi \int_{-a}^{b-a} (a^2 - y^2)dy = \pi a^2 b - \frac{\pi}{3}\left[(b-a)^3 + a^3\right]$$ $$= \pi a^2 b - \frac{1}{3}\pi(b^3 - 3ab^2 + 3a^2 b) = \frac{\pi}{3}\left(3ab^2 - b^3\right)$$ Now the volume of the complete sphere is $$\frac{4}{3}\pi a^3$$ so the computed volume is a fraction $$\frac{b^2(3a-b)}{4a^3}$$ When we put $$b = a$$ the fraction reduces to ½ as anticipated as this denotes the sphere is half full. When $$b = 2a$$ the fraction equals 1 corresponding to a completely full sphere

Mathematical behaviours	Marks
• writes down an appropriate integral…..	1
• …with the correct limits	1
• integrates correctly and inserts the limits	1
• simplifies the expression	1
• evaluates the correct fraction of the complete volume	1
• interprets correctly the case when $b = a$	1
• interprets correctly the case when $b = 2a$	1

Question 20 (a) **(3 marks)**

Solution	
$du = -\frac{1}{x}dx$ $\int \frac{1}{x\ln\left(\frac{1000}{x}\right)}dx = -\int \frac{du}{u}$ $= -\ln\lvert u\rvert + c$ $= -\ln\left\lvert \ln\left(\frac{1000}{x}\right)\right\rvert + c$	
Mathematical behaviours	**Marks**
• rewrites integral in terms of u	1
• determines correct antiderivative	1
• rewrites antiderivative in terms of x	1

Question 20(b) **(2 marks)**

Solution	
$\frac{d^2P}{dt^2} = r(k-P)\frac{dP}{dt} - rP\frac{dP}{dt}$ $= r\frac{dP}{dt}(k-2P)$ $\frac{d^2P}{dt^2} = 0 \quad \Rightarrow k-2P=0 \quad \Rightarrow \quad P = \frac{k}{2}$	
Mathematical behaviours	**Marks**
• differentiates implicitly to determine $\frac{d^2P}{dt^2}$	1
• establishes $P = \frac{k}{2}$	1

Question 20 (c) **(2 marks)**

Solution	
$\frac{d^2P}{dt^2} = -c\frac{dP}{dt} + c\ln\left(\frac{k}{P}\right)\frac{dP}{dt} = c\frac{dP}{dt}\left(\ln\left(\frac{k}{P}\right) - 1\right)$ $\frac{d^2P}{dt^2} = 0 \quad \Rightarrow \quad \ln\left(\frac{k}{P}\right) = 1$ $\Rightarrow \quad \frac{k}{P} = e$ $\Rightarrow \quad P = \frac{k}{e}$	
Mathematical behaviours	**Marks**
• differentiates implicitly to determine $\frac{d^2P}{dt^2}$ • establishes $P = \frac{k}{e}$	1 1

Question 20 (d) **(5 marks)**

Solution

$$\frac{dP}{dt} = 8\times10^{-5} P(1000 - P) \quad \Rightarrow \quad P = \frac{10^5}{100 + 900e^{-0.08t}}$$

$$t = 50 \quad \Rightarrow \quad P \approx 858$$

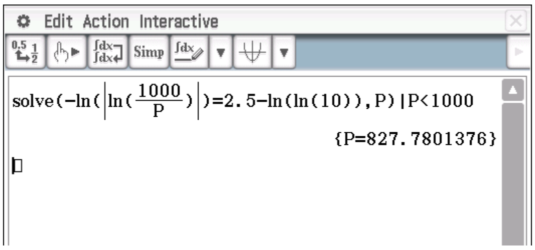
$$\frac{dP}{dt} = 0.05\ln\left(\frac{1000}{P}\right)P \quad \Rightarrow \quad \int \frac{dP}{P\ln\left(\frac{1000}{P}\right)} = \int 0.05\,dt$$

$$\Rightarrow \quad -\ln\left|\ln\left(\frac{1000}{P}\right)\right| = 0.05t + c$$

$$P_0 = 100 \quad \Rightarrow \quad c = -\ln|\ln 10|$$

$$t = 50, \quad -\ln\left|\ln\left(\frac{1000}{P}\right)\right| = 2.5 - \ln|\ln 10|$$

$$\Rightarrow \quad P \approx 828$$



The second model predicts a slightly smaller population.

Mathematical behaviours	Marks
• correct population equation using logistic equation	1
• calculates population size	1
• correct population equation using second model	1
• calculates population size	1
• compares predicted population sizes	1